# Common Fixed Point Theorems of Pair of Self Maps Satisfying Contractive Condition in G-Metric Space

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**Abstract**

In this paper we prove fixed point results of pair of self maps satisfying contractive condition involving maximum and minimum function in G-Metric space .

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## 1 Introduction

The metric fixed point theory is very important and useful in various areas such as variational inequalities,optimization and approximation theory .Many authors were introduced generalized structures of metric spaces.Gahler [1, 2] introduced generalized metric space called 2-metric space also B.C. Dhage [3, 4] obtained new structure known as D-Metric space.But in 2003,Z.Mustafa and B.Sims [5] found that there are some limitations in fundamental topological properties of D-metric space.And ,they [6] introduced a generalized Metric Space namely G-Metric space.For more detail information one can refer [7, 8, 9, 10].Ray (1976) [11] proved common fixed point theorem for the pair of self maps on complete Metric space X into itself.In 1977 B.Fisher [12] proved some results of common fixed points for pair of continuous maps which satisfies a contractive condition in terms of Maximum function.Then Z.Mustafa et. and all [7] proved some common fixed point theorems of pair of mapping satisfy contractive condition in terms of maximum function.

## 2 Preliminaries

**Definition 2.1.** *Let $X$ be a non empty set and $G : X^3 \to R^+$ which satisfies the following conditions*

1. $G(a, b, c) = 0$ *if* $a = b = c$ *i.e. every* $a, b, c$ *in* $X$ *coincides.*
2. $G(a, a, b) > 0$ *for every* $a, b, c \in X$ *s.t.* $a \neq b$
3. $G(a, a, b) \leq G(a, b, c)$, $\forall a, b, c \in X$ *s.t.* $c \neq b$
4. $G(a, b, c) = G(b, a, c) = G(c, b, a) = .........$
   *(symmetrical in all three variables)*
5. $G(a, b, c) \leq G(a, x, x) + G(x, b, c)$ , *for all a,b,c,x in X*
   *(rectangle inequality)*

   *Then the function G is said to be generalized metric or simply G-metric on X and the pair (X,G) is said to be G-metric space.*

**Example 2.2.** *Let $G : X^3 \to R^+$ s.t. $G(a, b, c) =$ perimeter of the triangle with vertices at a,b,c in $R^2$ , also by taking p in the interior of the triangle then rectangle inequality is satisfied and the function G is a G-metric on X.*

**Remark 2.3.** *G-metric space is the generalization of the ordinary metric space that is every G-metric space is (X,G) defines ordinary metric space $(X, d_G)$ by $d_G$ (a,b)=$G(a, b, b)+ G(a, a, b)$*

**Example 2.4.** *Let (X,d) be the usual metric space . Then the function $G : X^3 \to R^+$ defined by*

$$G(a, b, c) = max.\{d(a, b), d(b, c), d(c, a)\}$$

*for all a,b,c $\epsilon$ X is a G-metric space.*

**Definition 2.5.** *A G-metric space (X,G) is said to be symmetric if G(a,b,b)=G(a,a,b) for all a,b$\epsilon$X and if G(a,b,b)$\neq$ G(a,a,b) then G is said to be non symmetric G-metric space.*

**Example 2.6.** *Let X={x,y} and G : $X^3 \to R^+$ defined by G(x,x,x)=G(y,y,y)=0, G(x,x,y)=1, G(x,y,y)=2 and extend G to all of $X^3$ by symmetry in the variables. Then X is a G-metric space but It is non symmetric. since G(x,x,y) $\neq$ G(x,y,y)*

**Definition 2.7.** *Let (X,G) be a G-metric space, Let $\{a_n\}$ be a sequence of elements in X . The sequence $\{a_n\}$ is said to be G-convergent to a if*

$$lim_{m,n\to\infty}G(a,a_n,a_m)=0$$

*i.e for every $\epsilon > 0$ there is N s.t. $G(a,a_n,a_m) < \epsilon$ for all m,n $\geq N$ It is denoted as $a_n \to a$ or $lim_{n\to\infty}a_n = a$*

**Proposition 2.8.** *If (X,G) be a G-metric space. Then the following are equivalent*

1. *$\{a_n\}$ is G-convergent to a.*
2. *$G(a_n,a_n,a) \to 0$ as $n \to \infty$*
3. *$G(a_n,a,a) \to 0$ as $n \to \infty$*
4. *$G(a_m,a_n,a) \to 0$ as $m,n \to \infty$*

**Definition 2.9.** *Let (X,G) be a G-metric space a sequence $\{a_n\}$ is called G-Cauchy if , for each $\epsilon > 0$ there is an N $\epsilon I^+$ (set of positive integers) s.t.*

$$G(a_n,a_m,a_l) < \epsilon \; for \; all \; n,m,l \geq N$$

**Proposition 2.10.** *Let (X,G) be a G-metric space then the function G(a,b,c) is jointly continuous in all three of its variables.*

**Proposition 2.11.** *Let (X,G) be a G-metric space. Then, for any a,b,c,x in X it gives that*

1. $if\ G(a,b,c) = 0\ then\ a = b = c$

2. $G(a,b,c) \leq G(a,a,b) + G(a,a,c)$

3. $G(a,b,b) \leq 2G(b,a,a)$

4. $G(a,b,c) \leq G(a,x,c) + G(x,b,c)$

5. $G(a,b,c) \leq \frac{2}{3}(G(a,x,x) + G(b,x,x) + G(c,x,x))$

B.Fisher [12] proved following common fixed point theorem for pair of continuous mappings.

**Theorem 2.12.** *If $P$ and $Q$ are two continuous mappings of the complete metric space $X$ into itself s.t. $\rho(P^2x, Q^2y)) \leq cmax.\{\rho(Px,Qy), \rho(x,y)\}$ for all $x, y \in X$, where $0 \leq c < 1$. Then $P$ and $Q$ have a unique fixed point $z$.*

Z.Mustafa [14] proved a fixed point theorem in G-metric space which is a generalization of Banach contraction principle in G-metric space.

**Theorem 2.13.** *If $(X,G)$ be a complete G-Metric space and $F$ be a self mapping of $X$ which satisfies the following condition for all $a,b,c \in X$ $G(Fa,Fb,Fc) \leq \alpha G(a,b,c)$ for $0 \leq 1$. Then $F$ has a unique fixed point.*

Z.Mustafa et. and all [7] proved following theorem.

**Theorem 2.14.** *Let $(X,G)$ be a complete G-metric space, and $F$ be a self map which satisfy the condition*

$$G(F(a),F(b),F(c)) \leq \alpha max \begin{Bmatrix} G(a,F(a),F(a)) \\ G(b,F(b),F(b)) \\ G(c,F(c),F(c)) \end{Bmatrix}$$

*or*

$$G(F(a), F(b), F(c)) \leq \alpha max \begin{Bmatrix} G(a, a, F(a)) \\ G(b, b, F(b)) \\ G(c, c, F(c)) \end{Bmatrix}$$

*for all* $a, b, cin\ X$ *,for* $\alpha \in [0, 1)$ *.Then* $F$ *has a unique fixed point.*

Z.Mustafa and Hameed Obiedat [13] proved following common fixed point theorem.

**Theorem 2.15.** *Let* $(X, G)$ *be a complete G-metric space, and Let* $F : X \to X$ *be a map which satisfies*

$$G(F(a), F(b), F(c)) \leq \beta G(a, b, c) + \alpha\ max. \begin{Bmatrix} G(a, F(a), F(a)), G(b, F(b), F(b)) \\ G(c, F(c), F(c)) \end{Bmatrix}$$

*for all* $a, b, c \in X$ *,where* $0 \leq \beta + \alpha < 1$ *.Then* $F$ *has a unique fixed point (say v), and F is G-continuous at* $v$ *.*

## 3 Main Result

Now we prove result of common fixed point theorem for pair of self maps for following contraction.

**Theorem 3.1.** *If* $(X, G)$ *be a complete G-Metric space.If the pair of self maps* $P, Q$ *satisfy*

$$\begin{aligned} max \begin{Bmatrix} G(P(a), Q(b), Q(b)), \\ G(Q(a), P(b), P(b)) \end{Bmatrix} &\leq \alpha_1\ G(a, b, b) \\ &+ \alpha_2\ min \begin{Bmatrix} G(a, Q(b), Q(b)) + G(b, P(a), P(a))) \\ G(a, P(b), P(b)) + G(b, Q(a), Q(a)) \end{Bmatrix} \\ &+ \alpha_3\ min \begin{Bmatrix} G(a, P(a), P(a)) + G(b, Q(b), Q(b)), \\ G(a, Q(a), Q(a)) + G(b, P(b), P(b)) \end{Bmatrix} \end{aligned} \tag{3.1}$$

*for all $a, b$ in $X$, where $\alpha_1, \alpha_2, \alpha_3 \geq 0$ s.t. $\alpha_1 + 2\alpha_2 + 2\alpha_3 < 1$. Then $P$ and $Q$ have a unique common fixed point in $X$.*

*Proof.* Let $a_0$ be an arbitrary element of X.We define a sequence $\{a_n\}$ by

$$a_n = \begin{cases} P(a_{n-1}), & if\ n\ is\ odd \\ Q(a_{n-1}), & if\ n\ is\ even \end{cases}$$

If $n \in I^+$ be an odd positive integer then by using (3.1), we have

$$\begin{aligned} G(a_n, a_{n+1}, a_{n+1}) &= G(P(a_{n-1}), Q(a_n), Q(a_n)) \\ &\leq max. \begin{Bmatrix} G(P(a_{n-1}), Q(a_n), Q(a_n)) \\ G(Q(a_{n-1}), P(a_n), P(a_n)) \end{Bmatrix} \\ &\leq \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2\, min \begin{Bmatrix} G(a_{n-1}, Q(a_n), Q(a_n)) + G(a_n, P(a_{n-1}), P(a_{n-1})) \\ G(a_{n-1}, P(a_n), P(a_n)) + G(a_n, Q(a_{n-1}), Q(a_{n-1})) \end{Bmatrix} \\ &+ \alpha_3\, min \begin{Bmatrix} G(a_{n-1}, P(a_{n-1}), P(a_{n-1})) + G(a_n, Q(a_n), Q(a_n)) \\ G(a_{n-1}, Q(a_{n-1}), Q(a_{n-1})) + G(a_n, P(a_n), P(a_n)) \end{Bmatrix} \end{aligned}$$

Thus,

$$\begin{aligned} G(a_n, a_{n+1}, a_{n+1}) &\leq \alpha_1\, G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2\, \{G(a_{n-1}, Q(a_n), Q(a_n)) + G(a_n, P(a_{n-1}), Q(a_{n-1}))\} \\ &+ \alpha_3\, \{G(a_{n-1}, P(a_{n-1}), P(a_{n-1})) + G(a_n, Q(a_n), Q(a_n))\} \\ &= \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2\, \{G(a_{n-1}, a_{n+1}, a_{n+1}) + G(a_n, a_n, a_n)\} \\ &+ \alpha_3\, \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \\ &\leq \alpha_1\, G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2\, \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \\ &+ \alpha_3\, \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \end{aligned}$$

This gives, $G(a_n, a_{n+1}, a_{n+1}) \leq \frac{\alpha_1+\alpha_2+\alpha_3}{1-\alpha_2-\alpha_3} G(a_{n-1}, a_n, a_n)$

If $n \in N$ is an even then by using (3.1), we get

$$\begin{aligned} G(a_n, a_{n+1}, a_{n+1}) &= G(Q(a_{n-1}), P(a_n), P(a_n)) \\ &\leq max \begin{Bmatrix} G(P(a_{n-1}), Q(a_n), Q(a_n)) \\ G(Q(a_{n-1}), P(a_n), P(a_n)) \end{Bmatrix} \\ &\leq \alpha_1\ G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2\ min \begin{Bmatrix} G(a_{n-1}, Q(a_n), Q(a_n)) + G(a_n, P(a_{n-1}), P(a_{n-1})) \\ G(a_{n-1}, P(a_n), P(a_n)) + G(a_n, Q(a_{n-1}), Q(a_{n-1})) \end{Bmatrix} \\ &+ \alpha_3\ min \begin{Bmatrix} G(a_{n-1}, P(a_{n-1}), P(a_{n-1})) + G(a_n, Q(a_n), Q(a_n)) \\ G(a_{n-1}, Q(a_{n-1}), P(a_{n-1})) + G(a_n, P(a_n), P(a_n)) \end{Bmatrix} \\ &\leq \alpha_1\ G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2\ \{G(a_{n-1}, P(a_n), P(a_n)) + G(a_n, Q(a_{n-1}), Q(a_{n-1}))\} \\ &+ \alpha_3\ \{G(a_{n-1}, Q(a_{n-1}), Q(a_{n-1})) + G(a_n, P(a_n), P(a_n))\} \end{aligned}$$

$$\begin{aligned} G(a_n, a_{n+1}, a_{n+1}) &\leq \alpha_1\ G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2\ \{G(a_{n-1}, a_{n+1}, a_{n+1}) + G(a_n, a_n, a_n)\} \\ &+ \alpha_3\ \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \\ &\leq \alpha_1\ G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2\ \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \\ &+ \alpha_3\ \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \end{aligned}$$

which gives that, $G(a_n, a_{n+1}, a_{n+1}) \leq \frac{\alpha_1+\alpha_2+\alpha_3}{1-\alpha_2-\alpha_3} G(a_{n-1}, a_n, a_n)$ for any positive integer $n$, We have

(3.2) $$G(a_n, a_{n+1}, a_{n+1}) \leq \frac{\alpha_1+\alpha_2+\alpha_3}{1-\alpha_2-\alpha_3} G(a_{n-1}, a_n, a_n)$$

Let $\beta = \frac{\alpha_1+\alpha_2+\alpha_3}{1-\alpha_2-\alpha_3}$, $\therefore 0 \leq \beta < 1$ as $\alpha_1, \alpha_2, \alpha_3 \geq 0$ and $\alpha_1 + 2\alpha_2 + 2\alpha_3 < 1$ with this (3.2) becomes

(3.3) $$G(a_n, a_{n+1}, a_{n+1}) \leq \beta G(a_{n-1}, a_n, a_n)$$

by using relation (3.3) repeatedly, we get

$$G(a_n, a_{n+1}, a_{n+1}) \leq \beta^n G(a_0, a_1, a_1) \tag{3.4}$$

By using rectangular inequality repeatedly and using (3.4), we get, for all $n, m \in N$ s.t. $m > n$

$$\begin{aligned} G(a_n, a_m, a_m) &\leq G(a_n, a_{n+1}, a_{n+1}) + G(a_{n+1}, a_{n+2}, a_{n+2}) \\ &+ G(a_{n+2}, a_{n+3}, a_{n+3}) + \ldots.. + G(a_{m-1}, a_m, a_m) \\ &\leq (\beta^n + \beta^{n+1} + \ldots.. + \beta^{m-1})G(a_0, a_1, a_1) \\ &\leq \frac{\beta^n}{1-\beta} G(a_0, a_1, a_1) \end{aligned}$$

taking limit as $n, m \to \infty$ , we get $lim\ G(a_n, a_m, a_m) = 0$, since $lim_{n\to\infty} \frac{\beta^n}{1-\beta} = 0$ $\therefore$ limit of R.H.S. is 0. $\therefore$ $\{a_n\}$ is a G-Cauchy sequence, since X is G-complete. $\therefore$ there exists some $v \in X$ s.t. $\{a_n\}$ converges to $v \in X$. Again by using rectangle inequality and by equation (3.1), we get

$$\begin{aligned} G(v, Q(v), Q(v)) &\leq G(v, a_{2n+1}, a_{2n+1}) + G(a_{2n+1}, Q(v), Q(v)) \\ &= G(v, a_{2n+1}, a_{2n+1}) + G(P(a_2n), Q(v), Q(v)) \\ &\leq G(v, a_{2n+1}, a_{2n+1}) + max \begin{Bmatrix} G(P(a_{2n}), Q(v), Q(v)), \\ G(Q(a_{2n}), P(v), P(v)) \end{Bmatrix} \\ &\leq G(v, a_{2n+1}, a_{2n+1}) + \alpha_1\ G(a_{2n}, v, v) \\ &+ \alpha_2\ min \begin{Bmatrix} G(a_{2n}, Q(v), Q(v)) + G(v, P(a_{2n}), P(a_{2n})), \\ G(a_{2n}, P(v), P(v)) + G(v, Q(a_{2n}), Q(a_{2n})) \end{Bmatrix} \\ &+ \alpha_3\ min \begin{Bmatrix} G(a_{2n}, P(a_{2n}), P(a_{2n})) + G(v, Q(v), Q(v)), \\ G(a_{2n}, Q(a_{2n}), Q(a_{2n})) + G(v, P(v), P(v)) \end{Bmatrix} \\ &\leq G(v, a_{2n+1}, a_{2n+1}) + \alpha_1\ G(a_{2n}, v, v) \\ &+ \alpha_2\ \{G(a_{2n}, Q(v), Q(v)) + G(v, P(a_{2n}), P(a_{2n}))\} \\ &+ \alpha_3\ \{G(a_2n, P(a_{2n}), P(a_{2n})) + G(v, Q(v), Q(v))\} \end{aligned}$$

Thus, we have

$$\begin{aligned} G(v, Q(v), Q(v)) &\leq G(v, a_{2n+1}, a_{2n+1}) + \alpha_1 \, G(a_{2n}, v, v) \\ &+ \alpha_2 \, \{G(a_{2n}, Q(v), Q(v)) + G(v, a_{2n+1}, a_{2n+1})\} \\ &+ \alpha_3 \, \{G(a_{2n}, a_{2n+1}, a_{2n+1}) + G(v, Q(v), Q(v))\} \end{aligned}$$

taking limit as $n \to \infty$, and given that the function G is continuous on its variables , $\therefore$ we have $G(v, Q(v), Q(v)) \leq (\alpha_1 + \alpha_2) G(v, Q(v), Q(v))$ But, $0 \leq (\alpha_1 + \alpha_2) < 1$, $\therefore G(v, Q(v), Q(v)) \leq G(v, Q(v), Q(v))$, which is impossible. $\therefore G(v, Q(v), Q(v)) = 0$ $\therefore$ This gives $Q(v) = v$ .In the same way, we can prove that $P(v) = v$ . $\therefore$ $u$ is a common fixed point of P and Q. To prove uniqueness, if possible suppose that there exists some $u \in X$ s.t. P(u)=Q(u)=u. Then

$$\begin{aligned} G(v, u, u) &= G(P(v), Q(u), Q(u)) \\ &\leq max \begin{Bmatrix} G(P(v), Q(u), Q(u)), \\ G(Q(v), P(u), P(u)) \end{Bmatrix} \\ &\leq \alpha_1 G(v, u, u) \\ &+ \alpha_2 \, min \begin{Bmatrix} G(v, Q(u), Q(u)) + G(u, P(v), P(v)), \\ G(v, P(u), P(u)) + G(u, Q(v), Q(v)) \end{Bmatrix} \\ &\leq \alpha_3 \, min \begin{Bmatrix} G(v, P(v), P(v)) + G(u, Q(u), Q(u)), \\ G(v, Q(v), Q(v)) + G(u, P(u), P(u)) \end{Bmatrix} \\ &= \alpha_1 G(v, u, u) \\ &+ \alpha_2 \{G(v, u, u) + G(u, v, v)\} \\ &+ \alpha_3 \{G(v, v, v) + G(u, u, u)\} \end{aligned}$$

Which gives,

$$G(v, u, u) \leq \frac{\alpha_2}{1 - \alpha_1 - \alpha_2} G(u, v, v)$$

Similarly, we get

$$G(u, v, v) \leq \frac{\alpha_2}{1 - \alpha_1 - \alpha_2} G(v, u, u)$$

$\therefore$ from above two inequalities,we have

$$G(v,u,u) \leq (\frac{\alpha_2}{1-\alpha_1-\alpha_2})^2 G(v,u,u)$$

Since $0 \leq \frac{\alpha_2}{1-\alpha_1-\alpha_2} < 1$ $\therefore$ this gives $u = v$. □

**Theorem 3.2.** *If $(X,G)$ be a complete G-Metric space.If the pair of self maps $P,Q$ satisfy*

$$\max\begin{Bmatrix} G(P(a),Q(b),Q(b)), \\ G(Q(a),P(b),P(b)) \end{Bmatrix} \leq \alpha_1 G(a,b,b) + \alpha_2 \min\begin{Bmatrix} G(a,a,Q(b)) + G(b,b,P(a))) \\ G(a,a,P(b)) + G(b,b,Q(a)) \end{Bmatrix} + \alpha_3 \min\begin{Bmatrix} G(a,a,P(a)) + G(b,b,Q(b)), \\ G(a,a,Q(a)) + G(b,b,P(b)) \end{Bmatrix} \tag{3.5}$$

*for all $a,b$ in $X$ , where $\alpha_1,\alpha_2,\alpha_3 \geq 0$ s.t. $\alpha_1 + 2\alpha_2 + 2\alpha_3 < 1$ . Then $P$ and $Q$ have a unique common fixed point in X.*

*Proof.* Let $a_0$ be an arbitrary element of X.We define a sequence $\{a_n\}$ by

$$a_n = \begin{cases} P(a_{n-1}), & if\ n\ is\ odd \\ Q(a_{n-1}), & if\ n\ is\ even \end{cases}$$

Then by similar procedure used in last theorem (3.1), we have for any $n \in I^+$

$$G(a_n, a_{n+1}, a_{n+1}) \leq \beta^n G(a_0, a_1, a_1) \tag{3.6}$$

then by using rectangle inequality and equation (3.6) , we get, for all $n,m \in I^+ , m > n$

$$\begin{aligned} G(a_n,a_m,a_m) &\leq G(a_m,a_{m-1},a_{m-1}) + G(a_{m-1},a_{m-2},a_{m-2}) \\ &+ G(a_{m-2},a_{m-3},a_{m-3}) + ..... + G(a_{n+1},a_n,a_n) \\ &\leq (\beta^n + \beta^{n+1} + ..... + \beta^{m-1})G(a_0,a_1,a_1) \\ &\leq \frac{\beta^n}{1-\beta} G(a_0,a_1,a_1) \end{aligned}$$

$\therefore \{a_n\}$ be a Cauchy sequence.And by completeness of G-metric space, there exists $v \in X$ s.t. $\{a_n\}$ is G-convergent to $v$.As in last proof we prove that,

$$G(v, v, Q(v)) \leq (\alpha_1 + \alpha_2)G(v, v, Q(v))$$

Thus required conclusion follows from same argument used in last theorem. □

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