ISBN No.978-93-82995-25-8

Advanced Studies in Pure Science and Applied Science

श्री संत जनाबाई शिक्षण संस्था
विद्या विनयेन शोभते
कला, वाणिज्य व विज्ञान
कनिष्ठ व वरिष्ठ महाविद्यालय

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ISBN No. 978-93-82995-25-8

Published by:

Anuradha Publications

Cidco-Nanded

Publication Year: 2018-19

Price- Rs. 250/-

Printed by

Gurukrupa Offset,

Near Police Station, Gangakhed

Typesetting by:

Simran Computers

Gangakhed Dist.Parbhani

Cover Designby:

Mr. Imran K. Mohammad

ISBN No.978-93-82995-25-8

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Results on Kanan Fixed Point Theorem in Generalized Metric Space (g.m.s.)

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Abstract:

In this paper we prove a unique fixed point theorem of Kanan map in Generalized Metric space using uniformly locally contractive map.

Keywords:

Generalized Metric Space, Contraction mapping, fixed point, orbitally complete mapping, and Uniformly Locally Contractive map.

1. Introduction

In 1922 S.Banach proved a Contraction mapping principle [3] states that S has a unique fixed point.Recently in 2000 a very interesting notion of generalized metric space was introduced by Branciari [2]. Branciari generalized Banach fixed point result in Generalized metric space(g.m.s.). In 2002 P.Das. [5] generalized fixed point result in generalized metric space (g.m.s.).Later in 2009 Dorel Mihet[7] generalized Kanan fixed point theorem in this space.

2. Some preliminaries

Definition 2.1 [2] :- Let X be any non-empty set and $d: X \times X \to R^+$ be a self map. Then d *is* said to be as Generalized metric if it satisfies following

Postulates for all $x, y, z \in X$

$(i) d(x, y) = 0$ iff $x = y$

$(ii) d(x, y) = d(y, x)$ (symmetry)

$(iii) d(x, y) \leq d(x, z) + d(z, w) + d(w, y)$ (rectangular inequality)

A set X equipped with metric d is said to be a generalized metric space or (g.m.s.)

Remark:- Every metric space is a generalized metric space but the converse may not always be true.

Definition 2.4 [6] :- Let T be a mapping of g.m.s. (X, d) into itself is called as T-Orbitally complete iff every Cauchy Sequence

$\{x_n\} \subseteq \{x, Tx, T^2x, T^3x, ...\}$ for $x \in X$ Converges in X itself.

Definition 2.5:- $T: X \to X$ is called $(\in, \mu)$ Uniformly locally contractive if it is locally contractive at all points of $x \in X$ and $\in, \mu$ do not depend on x i.e.

$d(x, y) < \in \Rightarrow d(Tx, Ty) < \mu\, d(x, y)$ for all $x, y \in X$

Kanan generalized a fixed point result in a g.m.s. [3] stated as

Theorem 3.1 Let X be a Complete metric Space and $T: X \rightarrow X$ is a mapping such that,

$d(Tx,Ty) \leq \beta\, [d(x,Tx) + d(y,Ty)]$, for all $x, y \in X$.

Where $\beta \in\ ]0,1[$.Then T has a Unique fixed point in X.

By taking concept of F-orbitally Complete maps then a slight generalization of Theorem 3.1 is given by

Theorem 3.2 [7] If (X,d) be a generalized metric space and $S: X \rightarrow X$ is a mapping such that,

$d(Sx,Sy) \leq \beta\, [d(x,Sx) + d(y,Sy)]$ (2)

for all $x, y \in X$

Where $\beta \in\ ]0,1[$. If X is S-orbitally complete. Then S has a Unique fixed point in X.

We generalize Theorem 3.1 in orbitally complete Generalized metric space.

3. Main Result

Theorem 3.3 Let (X,d) be a generalized metric space, and the mapping $F: X \rightarrow X$ is uniformly locally contractive which satisfies (2).

Then F has a Unique fixed point in X.

Proof :- Let x_0 be any point in X . Let F(x_0)=x_1 .

If x_1 and x_0 coincides then F(x_0)=x_1 . It shows that x_0 is a fixed Point of *F*. And the theorem is obviously proved.

Let us suppose that x_1 and x_0 are distinct i.e. $x_0 \neq x_1$

consider $F(x_1) = x_2, F(x_2) = x_3, \ldots\ldots,$

$F(x_n) = x_{n+1} = F^{n+1}x_0$ and $x_{n+1} \neq x_n$ for n=0,1,2,.......

In this way we have a sequence $\{x_n\}$ in this manner.

Now consider

$d(x_{n,}x_{n+1}) = d(Fx_{n-1}, Fx_n)$

$$\leq \beta[d(x_{n-1}, Fx_{n-1}) + d(x_n, Fx_n)]$$

($\because$ by inequality (2))

$$\leq \beta\, [d(x_{n-1,}x_n) + d(x_{n,}x_{n+1})]$$

$$\leq \frac{\beta}{1-\beta}\, d(x_{n-1,}x_n)$$

Let us suppose that x_0 is not a periodic point, and if $x_n = x_0$, then

We have,

ISBN No.978-93-82995-25-8

$d(x_0, Fx_0) = d(x_n, Fx_n) = d(F^n x_0, F^{n+1} x_0)$

$$\le \frac{\beta}{1-\beta} d(F^{n-1}x_0, F^n x_0)$$

$$\le (\frac{\beta}{1-\beta})^2 d(F^{n-1}x_0, F^{n-1}x_0)$$

$\le$...............

$\le$...............

$$\le (\frac{\beta}{1-\beta})^n d(x_0, Fx_0) \qquad (3)$$

Let $\alpha = \dfrac{\beta}{1-\beta}$ $\therefore$ $\alpha < 1$ and

$(1-\alpha^n)\, d(x_0, Fx_0) \le 0$

This gives $d(x_0, Fx_0) \le 0$

$\Rightarrow$ $Fx_0 = x_0$.It means x_0 is a fixed point of T.

Now relation (2) gives

$d(F^n x_0, F^{n+m} x_0) \le \beta\, [d(F^{n-1} x_0, F^n x_0) + d(F^{n+m-1} x_0, F^{n+m} x_0)$

$$\le \beta\, [\alpha^{n-1} d(x_0, Fx_0) + \alpha^{n+m-1} d(x_0, Fx_0)]$$

$\therefore$ $d(x_{n,} x_{n+m}) \to 0$, as n $\to \infty$

Hence $\{x_n\}$ is a Cauchy sequence in X. As X is a Complete $\therefore \exists$ a $p \in X$ s.t. $x_n \to p$

By using quadrilateral property of g.m.s.

We have

$d(F_p, p) \le d(F_p, F^n x_0) + d(F^n x_0, F^{n+1} x_0) + d(F^{n+1} x_0, p)$

$$\le \beta\, [d(p, F_p) + d(F^{n-1} x_0, F^n x_0)]$$

$$+ \alpha^n d(x_0, Fx_0) + d(F^{n+1} x_0, p)$$

$$\le \alpha\, d(F^{n-1} x_0, F^n x_0) + \frac{\alpha^n}{1-\beta} d(x_0, Fx_0)$$

$$+ \frac{1}{1-\beta} d(F^{n+1} x_0, p)$$

So taking limit as n $\to \infty$ and using the fact that,

$d(a_n, y) \rightarrow d(a, y)$ and $d(x, a_n) \rightarrow d(x, a)$

whenever $\{a_n\} \in X$ with $a_n \rightarrow a \in X$

This gives $p = Fp$.

i.e. p is a fixed point of F.

To prove uniqueness If possible suppose that $q \in X$ is another fixed point of F i.e $Fq=q$

Consider,

$$d(q,p) = d(Fq, Fp)$$

$$\leq \beta\, [d(q, Fq) + d(p, Fp)$$

$$\leq \beta\, [d(q,q) + d(p,p) = 0$$

$$\Rightarrow p = q.$$

Hence the Fixed point of F is Unique.

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